Atoms and Clusters
WS 11/12

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Problem Set 5

Questions

(Q.1) What determines the diffraction pattern for a single-slit and a grating?

(Q.2) Explain in simple physical terms why the diffraction orders of a grating occur at the same angles as the constructive interference between a pair of slits with the same spacing as those in the grating.

(Q.3) Why leads Doppler broadening to a Gaussian line shape but pressure broadening to a Lorentzian?

Problems

(5.1) **Sum rule**
For the sake of brevity, the Thomas-Reiche-Kuhn sum rule for the oscillator strengths $f_{ka}^{(N)}$,

$$\sum_k f_{ka}^{(N)} = N, \qquad f_{ka}^{(N)} = \frac{2m\omega_{ka}}{3\hbar}|\boldsymbol{R}_{ka}|^2, \qquad \boldsymbol{R}_{ka} = \sum_{i=1}^{N} \boldsymbol{r}_{ka}^{(i)}, \qquad \boldsymbol{r}_{ka}^{(i)} = \langle\psi_k|\boldsymbol{r}_i|\psi_a\rangle$$

was derived only for $N = 1$-electron atoms in the lecture. Show that it also holds for N-electron atoms.

(5.2) **Measurement of the van der Waals interaction with nano-fabricated grating**
Diffraction by a grating with slits of width a and spacing d gives an intensity distribution:

$$I = I_0 \left(\frac{\sin(Nud/2)}{\sin(ud/2)}\right)^2 \left(\frac{\sin(ua/2)}{ua/2}\right)^2 .$$

Here $u = 2\pi\sin\theta/\lambda_{dB}$ and the angle is defined in the Picture, λ_{dB} is the de Broglie wavelength.

An experimental observation of the diffraction of rare gas atoms from the grating showed that the intensity of the second order is $0.003I_0$ for helium and $0.05I_0$ for krypton. The difference in these values was ascribed to the van der Waals force, which is strongest for large atoms. Therefore a krypton atom on a trajectory that goes close to the sides of the slit feels a force that deflects it through a large angle or causes it to crash into the grating. These processes effectively reduce the slit width from a to $a - 2r$, where r is the typical van der Waals range. Estimate r for krypton atoms. Refer to a grating with $d = 2a = 100$nm.

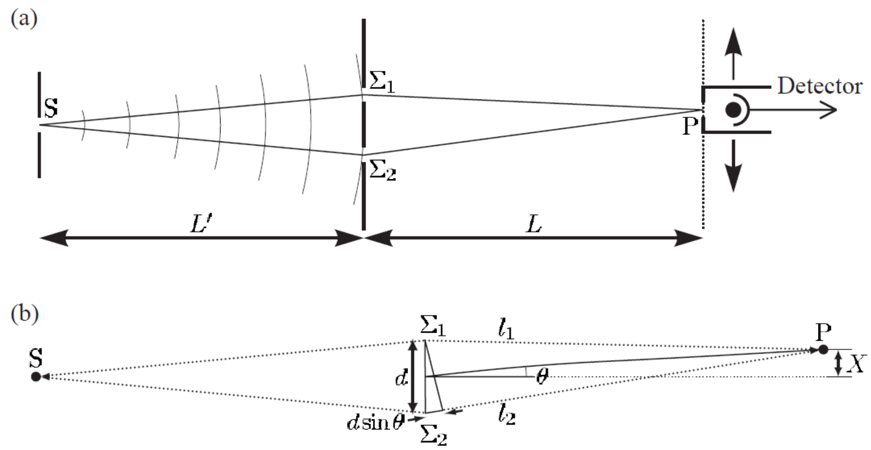

Figure 1: The apparatus for observing interference from double slits. The light diffracted by the source slit propagates through the two slits Σ_1 and Σ_2 and onto the plane P. The interference fringes can be seen with the eye (with the aid of a magnifying eyepiece if necessary), but to further the analogy with atom optics experiments the apparatus is drawn with a detector such as a photodiode or photomultiplier. A narrow slit in front of the detector gives good spatial resolution; this slit and the detector scan across the fringes, as indicated. The light comes from a lamp, or laser; in a matter-wave experiment an atomic oven creates a beam of atoms collimated by the source slit. (b) The difference in the distance from Σ_1 to P and from Σ_2 to P is $d \sin\theta$, where d is the slit separation. The angle θ and the distance X in the detection plane are related by $X = L \tan\theta$. The transverse distances are drawn greatly exaggerated for clarity; for the typical conditions the fringes have an angular separation of 2×10^{-3} rad.