Advanced Theoretical Physics
WS 09/10

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Seminar 5

Keywords:

Interpretations of Quantum Mechanics,
measurement, uncertainty relations

Questions:

1. Why do non-commuting observables fulfill an uncertainty relation?
2. Why do energy and time fulfill an uncertainty relation although time is not an operator and thus commutes with energy?

Assignment 5

(due November 16, 2009)

Commutators

5.1 Let $\hat{\boldsymbol{\ell}}$ be the angular momentum vector operator and $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{p}}$ position and momentum vector operators, respectively. Show that $[\hat{\boldsymbol{x}}^2, \hat{\boldsymbol{\ell}}] = [\hat{\boldsymbol{p}}^2, \hat{\boldsymbol{\ell}}] = [\hat{\boldsymbol{\ell}}^2, \hat{\boldsymbol{\ell}}] = 0$.

5.2 Show that for arbitrary operators $\hat{L}$, $\hat{M}$

$$[\hat{L}, \hat{M}^n] = \sum_{\lambda=0}^{n-1} \hat{M}^{\lambda}[\hat{L}, \hat{M}]\hat{M}^{n-\lambda-1}.$$

5.3 Show that for position and momentum operators $\hat{x}$, $\hat{p}$ one has

$$[\hat{p}^r, \hat{x}^s] = \frac{\hbar}{\mathrm{i}} r \sum_{\sigma=0}^{s-1} \hat{x}^{\sigma}\hat{p}^{r-1}\hat{x}^{s-\sigma-1} = \frac{\hbar}{\mathrm{i}} s \sum_{\rho=0}^{r-1} \hat{p}^{\rho}\hat{x}^{s-1}\hat{p}^{r-\rho-1}.$$

Matrix elements and sum rules

5.4 Calculate the matrix elements of $\dot{\hat{L}}$ (i.e., the operator describing the temporal change of the operator $\hat{L}$) expressed in terms of the matrix elements of $\hat{L}$ and $\partial\hat{L}/\partial t$ in energy representation (i.e., with energy eigenvectors $|E\rangle$ as basis vectors).

5.5 Show that for a particle of mass m in a potential

$$\langle E|\hat{p}|E'\rangle = \frac{\mathrm{i}m}{\hbar}(E - E')\langle E|\hat{x}|E'\rangle,$$

$$\langle E|\hat{F}|E'\rangle = \frac{\mathrm{i}}{\hbar}(E - E')\langle E|\hat{p}|E'\rangle.$$

Here, $\hat{F} = \dot{\hat{p}}$.

5.6 Show the sum rules

$$S_1 = \sum_a (E_a - E_n)|\langle E_a|\mathrm{e}^{\mathrm{i}k\hat{x}}|E_n\rangle|^2 = \frac{\hbar^2 k^2}{2m},$$

$$S_2 = \sum_a (E_a - E_n)|\langle E_a|\hat{x}|E_n\rangle|^2 = \frac{\hbar^2}{2m}.$$